Sound synthesis with Periodically Linear Time Varying Filters

Antonio Goulart, Marcelo Queiroz
Joseph Timoney, Victor Lazzarini

Computer Music Research Group - IME/USP - Brazil
Sound and Digital Music Technology Group - NUIM - Ireland

antonio.goulart@usp.br

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Motivations

- New synth sounds ▷
- Low computational cost
- Virtual Analog Oscillators
- Usage as audio effect

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- Low computational cost
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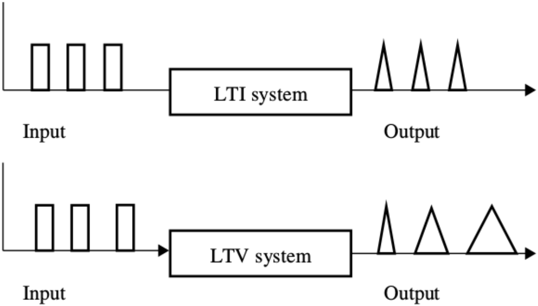
- The challenge: ▷

"When I first got some - I won't call it music - sounds out of a computer in 1957, they were pretty horrible. (...) Almost all the sequence of samples - the sounds that you produce with a digital process - are either uninteresting, or disagreeable, or downright painful and dangerous. **It's very hard to find beautiful timbres.**"

Max Mathews, 2010.

Contribution

LTV theory approach to distortion techniques



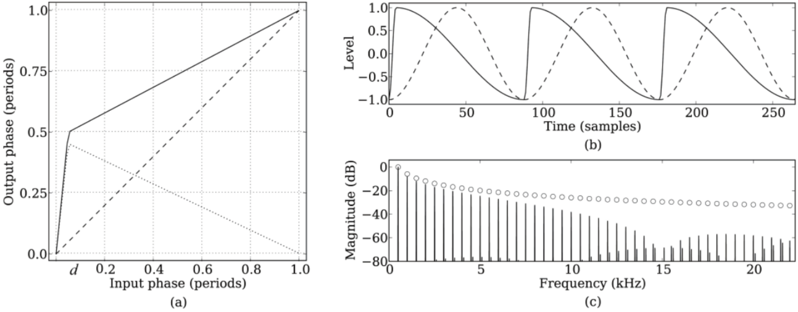
$$h(p, n) \quad H(z, n) \quad H(\omega, n)$$

Phaseshaping - US patent 4658691 Casio - CZ

Add a phase distortion function to the regular phase generator
Sawtooth: Inflection point on the regular (dashed) index

$$t + g(t) = \begin{cases} 0.5\frac{t}{d}, & 0 \le t \le d \\ 0.5\frac{t-d}{1-d} + 0.5, & d < t < 1 \end{cases}$$

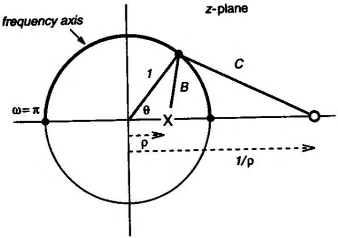
For $d = 0.05$



(a) (b) (c)

The allpass filter

$$H(z) = \frac{-a + z^{-1}}{1 - az^{-1}}$$



Flat magnitude response

Frequency dependent phase shift

$$\phi(\omega) = -\omega + 2\tan^{-1}\left(\frac{-a\sin(\omega)}{1 - a\cos(\omega)}\right)$$

Reverb, chorus, flanger, phaser, spectral delay

Amplitude modulation

$$\cos\left(2\pi f_c n\right)\cos\left(2\pi f_m n\right) = \frac{1}{2}\cos\left(2\pi f_c n + 2\pi f_m n\right) + \frac{1}{2}\cos\left(2\pi f_c n - 2\pi f_m n\right)$$

Jussi Pekonen, 2008

Coefficient-modulated first-order allpass filter as distortion effect

- Suggests the method for sound synthesis and audio effects
- Recall that classic PD is restricted to cyclic tables
- Derives stability condition

$$|m(n)| \leq 1 \quad \forall n$$

- Recommends appropriate values for $m(n)$
- Allpass Dispersion on low frequencies

$$\phi_{DC}(n) = \frac{1 - m(n)}{1 + m(n)}$$

J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki, 2009

Spectrally rich phase distortion sound synthesis using allpass filter

Time-varying allpass transfer function

$$H(z, n) = \frac{-m(n) + z^{-1}}{1 - m(n)z^{-1}}$$

Phase distortion

$$\phi(\omega, n) = -\omega + 2\tan^{-1}\left(\frac{-m(n)\sin(\omega)}{1 - m(n)\cos(\omega)}\right)$$

J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki

$$\phi(\omega, n) = -\omega + 2\tan^{-1}\left(\frac{-m(n)\sin(\omega)}{1 - m(n)\cos(\omega)}\right)$$

Knowing $\phi(\omega, n)$, use $tan(x) \approx x$,

$$m(n) = \frac{-(\phi(\omega, n) + \omega)}{2\sin(\omega) - (\phi(\omega, n) + \omega)\cos(\omega)}$$

J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki

Emulate the classic phase distortion technique

$$t + g(t) = \begin{cases} 0.5\frac{t}{d}, & 0 \le t \le d \\ 0.5\frac{t-d}{1-d} + 0.5, & d < t < 1 \end{cases}$$

Subtract linear phase from the phase distortion function

$$g(t) = \begin{cases} (\frac{1}{2} - d)\frac{t}{d}, & 0 \le t \le d \\ (\frac{1}{2} - d)\frac{1-t}{1-d} + 0.5, & d < t < 1 \end{cases}$$

J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki

Range for the allpass modulation should be $[-\omega, -\pi]$

$$\phi(\omega, t) = \frac{g(t)((1-2d)\pi - \omega)}{(1-2d)\pi} - (1-2d)\pi - \omega$$

Get the modulation function

$$m(n) = \frac{-(\phi(\omega, n) + \omega)}{2\sin(\omega) - (\phi(\omega, n) + \omega)\cos(\omega)}$$

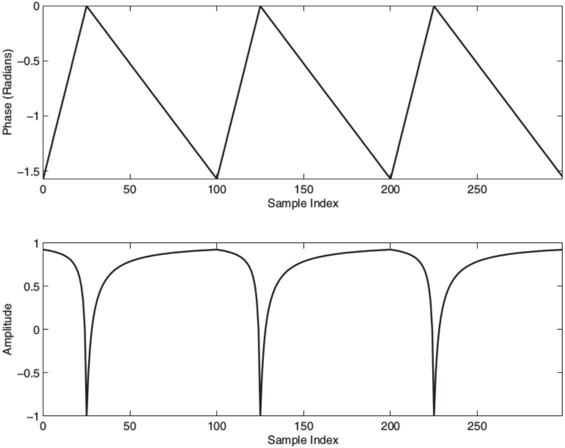
Implementation with difference equations

$$y(n) = x(n-1) - m(n)(x(n) - y(n-1))$$

▷

J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki

Phase distortion and coefficient modulation functions



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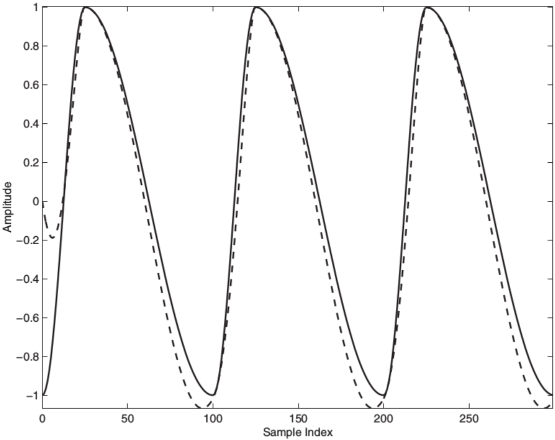
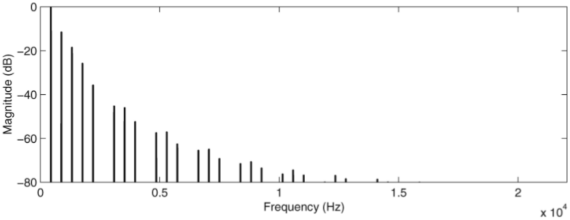
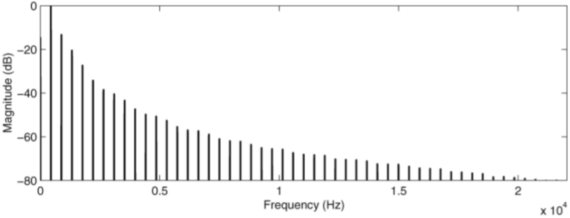
Outputs with classic PD (solid) and modulated allpass (dashed)



J.Timoney, V.Lazzarini, J.Pekonen, V.Valimaki

Classic PD and Modulated allpass spectra





Arbitrary distortion function

$$y(n) = 0.4\cos(f_0) + 0.4\cos\left(2f_0 - \frac{\pi}{3}\right) + 0.35\cos\left(3f_0 + \frac{\pi}{7}\right) + 0.3\cos\left(4f_0 + \frac{4\pi}{3}\right)$$

Shift it to the appropriate range

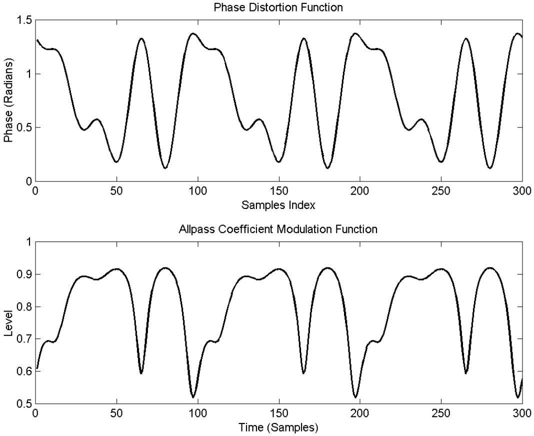
$$y_s(n) = -\frac{\pi}{2}\frac{(y(n)+1)}{2}$$

Create your own (:

▷

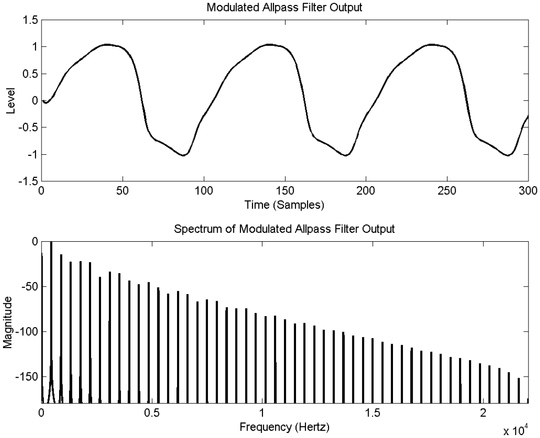
Arbitrary distortion function

Phase distortion and derived modulation functions



Arbitrary distortion function

Waveform and spectrum



J.Kleimola, V.Lazzarini, J.Timoney, V.Valimaki, 2009

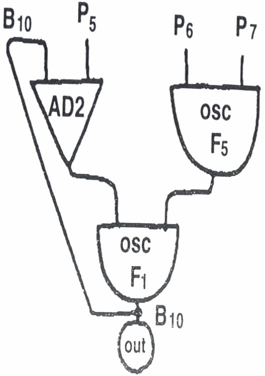
FeedBack Amplitude Modulation (FBAM)

Revisiting of an old idea by A.Layzer
tested by Risset in the catalogue ▷

Modulate oscillator amplitude using its output

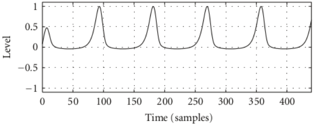
$y(n) = \cos(\omega_0 n)[1 + \beta y(n-1)]$

with $\omega_0 = 2\pi f_0$ and $y[0] = 0$



FeedBack Amplitude Modulation

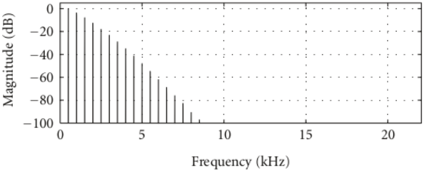
$$y(n) = \cos(\omega_0 n)[1 + y(n-1)]$$



$$y(n) = \cos(\omega_0 n) +$$
$$\cos(\omega_0 n)\cos(\omega_0[n-1]) +$$
$$\cos(\omega_0 n)\cos(\omega_0[n-1])\cos(\omega_0[n-2]) + ...$$

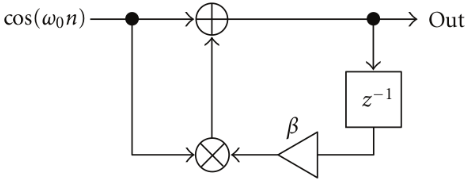
$$= \sum_{k=0}^{\infty}\prod_{m=0}^{k}\cos[\omega_0(n-m)]$$

$$\cos^2(p) = \frac{1}{2}(1+\cos(2p))$$
$$\cos^3(p) = \frac{1}{4}(3\cos(p) + cos(3p))$$



FeedBack Amplitude Modulation

LPTV interpretation



$y(n) = x(n) + \beta a(n) y(n-1)$

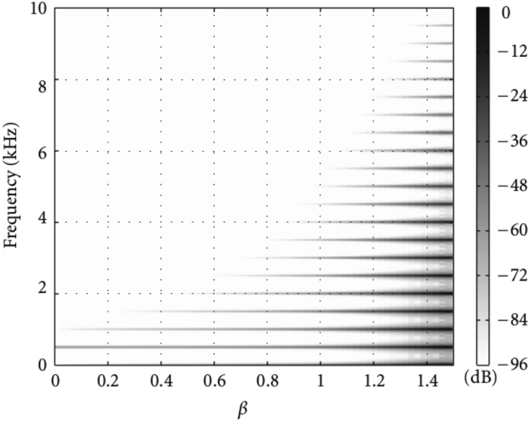
$x(n) = a(n) = \cos(\omega_0 n)$
in this case (but could be $\neq$)

1 pole coefficient modulated IIR $\rightarrow$ Dynamic PD

$\triangleright$

Feedback Amplitude Modulation

β similar to FM's modulation index

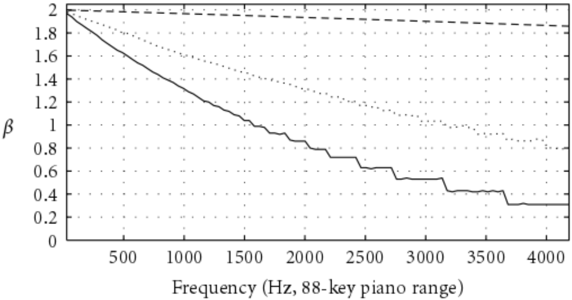


Feedback Amplitude Modulation

Stability condition

$$\left|\beta \prod_{m=1}^{N} \cos\left(\omega_0 m\right)\right| < 1$$

Aliasing before instability

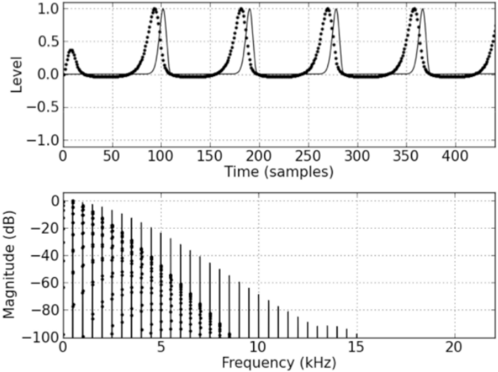


2nd order FBAM

Two previous outputs with individual βs

$$y(n) = \cos(\omega_0 n)[1 + \beta_1 y(n-1) + \beta_2 y(n-2)]$$

Narrower pulse and wider band ▹



Conclusions

- Reissue of a classic technique
- Different kind of implementation
- Input and modulation can be arbitrary signals

- Deeper investigation of LTV
- Studying 2nd and higher order systems stability

Thanks a lot!

antonio.goulart@usp.br

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Núcleo de Pesquisas
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